IP/BBSR/96-13
hep-th/9602144

M-Theory on Orientifolds of $K_3 \times S^1$

Alok Kumar and Koushik Ray
Institute of Physics,
Bhubaneswar 751 005, INDIA
email: kumar, koushik@iopb.ernet.in

February 7, 2008

Abstract

We present several Orientifolds of M-Theory on $K_3 \times S^1$ by additional projections with respect to the finite abelian automorphism groups of K_3. The resulting models correspond to anomaly free theories in six dimensions. We construct explicit examples which can be interpreted as models with eight, four, two and one vector multiplets and $N = 1$ supersymmetry in six dimensions.

M-Theory[1, 2, 3, 4, 5, 6, 7, 8, 9], believed to be a candidate for the unification of all string theories, is at present a focus of attention. At low energies, this theory is represented by the eleven-dimensional supergravity. It was already shown earlier, that the eleven-dimensional supergravity is the strong coupling limit of the ten dimensional type IIA string theory[1]. Recent interest in the subject was generated by the fact that the compactification of these theories to ten dimensions on an orientifold[10, 11, 12] of S^1 gave rise to the $E_8 \times E_8$ heterotic string theory in ten dimensions[4]. In proving this equivalence, the anomaly cancellations for the ten dimensional $N = 1$ supersymmetric theories plays an important role[13]. This is due to the fact that, in the absence of a complete knowledge of this theory, an explicit construction of the orientifolds is not possible. More recently, compactifications of M-Theory to six dimensions have also been studied. Its T^5/Z_2 orientifold gave rise to an anomaly free $N = 2$ supersymmetric theory in six dimensions with 21 tensor multiplets[5, 6]. Once again the requirements of the anomaly cancellation[14] played a major role in determining the complete spectrum of this theory. It was shown that the fixed points of the torus degenerate into pairs. As a result, the twisted sectors contribute only sixteen extra tensor multiplets, instead of thirty two.

The orientifolds of type II string theories have also been examined [15, 16, 17]. In [17] construction of a new chiral string theory in six dimesnions, through an orientifold compactification of the type IIB strings on K_3, has been presented. It was shown that such a compactification gives rise to an $N = 1$ supersymmetric theory in six dimensions with anomaly free particle content.

A particle spectrum, consisting of 9 tensor, 8 vector and 20 hypermultiplets and $N = 1$ supersymmetry was obtained in six dimesnions by Sen[9] for M-Theory compactified on an orientifold of $K_3 \times S^1$. In this case, like in the ten dimensional one, the twisted sector states can not be obtained by a direct M-Theory calculation. However, Sen was able to determine the spectrum in the twisted sector by comparing the nature of the fixed points for $K_3 \times S^1$ with those for T^5. It was argued that the physics near

the fixed points in the case of $(K_3 \times S^1)/Z_2$ is identical to that for T^5/Z_2. As a result, the contribution of the twisted sector states are identical in the two cases. This fact will also be utilized in our case below. Since the extra Z_2's that we apply act freely and keep the supersymmetry intact, the number of fixed points remains unchanged. Their contribution to the field content also does not change, since they still come as tensor multiplets of the chiral $N = 2$ algebra[6, 9] due to the supersymmetry preserving nature of the extra projections[20]. This is further confirmed by the fact that we are able to obtain anomaly free combination of fields in all our examples.

In this article, we present new examples of orientifold compactifications of M-Theory by further orbifolding of the models in [9] with respect to the finite abelian automorphism groups of K_3[18]. In particular, we present several $N = 1$ supersymmetric examples with different number of vector multiplets. The orbifolds of K_3 for the case of type IIA string compactification was discussed in [19, 20]. These orbifolds provided examples of the dual pairs of the heterotic string theories in dimensions six and less with maximal supersymmetry, but with lower rank gauge groups. In this article we focus our attention on the Z_2^k orbifolds discussed in [20]. In our case, we combine one of these Z_2 actions with another operation which changes the sign of the eleven dimensional 3-form fields as well as the elven-dimensional coordinate x^{10}[9]. As a result we are able to construct several models with $N = 1$ supersymmetry, but with different number of vector multiplets. Extra Z_2 symmetries, in our case act freely. To avoid new fixed points, we combine these Z_2's with the translation symmetries along some of the compactified directions. Consequently, like in [20], our result is strictly valid only in dimensions less than six. The lower dimesnional spectra can however be seen to arise from the corresponding six dimensional ones, with new anomaly free combinations, in a straightforward way.

We now begin by describing the Z_2^k $(k = 1, 2, 3, 4)$ orbifolds presented in [20]. The action of the symmetries is represented by k Z_2 generators, denoted by g_i $(i = 1, .., k)$. Under these Z_2's all the three self-dual two forms and three of the nineteen anti-self-

dual three forms are invariant. On the remaining sixteen anti-self-dual two forms it acts as:

$$g_1 : \Big((-1)^8, 1^8\Big)\,, \tag{1}$$

$$g_2 : \Big((-1)^4, 1^4, (-1)^4, 1^4\Big)\,, \tag{2}$$

$$g_3 : \Big((-1)^2, 1^2, (-1)^2, 1^2, (-1)^2, 1^2, (-1)^2, 1^2\Big)\,, \tag{3}$$

$$g_4 : (-1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1)\,, \tag{4}$$

with superscripts on 1 denoting the repeated entries. Individually, as will be described below, under any of these Z_2's 34 of the 58 K_3 moduli fields remain invariant. Similarly, 14 of the 22 two-forms on K_3 are even under Z_2's and 8 are odd. When more than one of these discrete symmetries is used to mod out the original theory, the resulting spectrum is determined by taking the intersection of the individual ones.

Above discrete symmetries are now used to obtain the untwisted sector of the spectrum when M-Theory, with low energy spectrum consisting of a graviton G_{MN} and a third rank antisymmetric tensor A_{MNP} in eleven dimensions, is compactified on an orientifold of $K_3 \times S^1$. For this purpose, the first Z_2 in equation (1) is combined with an operation $A_{MNP} \rightarrow -A_{MNP}$ and $x^{10} \rightarrow -x^{10}$. Here we have used the notation that $(M, N) = (0, ..., 10)$ and $(\mu, \nu) = (0, .., 5)$. The rest of the g_i's act in the same way as in [20]. We would once again like to point out that projections in [20] with respect to Z_2's in equations (1)-(4) keeps the supersymmetries unbroken. As a result, further orbifolding, by g_i's, of the resulting $N = 1$ theory also keeps the supersymmetry intact.

We now present the counting of the surviving degrees of freedom when several g_i's in equations (1)-(4) are applied on G_{MN} and A_{MNP}. First, the surviving degrees of freedom for G_{MN} is determined by examining the local structure of the moduli space. Since the number of supersymmetries for the Z_2 projections by g_i's remain

unchanged, the local structure of the moduli space for the compactifications on the above orbifolds of K_3 has the form [20]:

$$\mathcal{M} = \frac{SO(20-r, 4; R)}{SO(20-r; R) \times SO(4; R)}, \tag{5}$$

where r is the reduction in the rank of the gauge group from the maximal number 24, which is equal to the one for the toroidal compactification of the heterotic string. Such reductions in the ranks of the gauge group, for the orbifolds of K_3 described above, can now be studied in the type IIA theory. We will use this information to determine the number r for the action of the various combinations of g_i's.

All the 24 gauge fields in the K_3 compactification of type IIA theory originate in the Ramond-Ramond (R-R) sector. Since for K_3, only nonzero Betti numbers are $b_0 = b_4 = 1$ and $b_2 = 22$, one of the gauge fields in six dimensions originates from its ten dimensional counterpart A_M. Three-form field components $A_{\mu mn}$ give rise to 22 gauge fields and the dualization of $A_{\mu\nu\rho}$ gives the remaining one. Here (m, n) denote the indices on K_3. An observation of the form of g_i's in equations (1)-(4) now gives the values:

(i) $r = 8$ for the action of g_1,

(ii) $r = 12$ for the action of g_1 and g_2 together,

(iii) $r = 14$ for the action of g_1, g_2 and g_3, and finally

(iv) $r = 15$, when all the four g_i's in equations (1)-(4) are applied.

The reduction in the number of gauge fields follows directly by counting the number of 2-forms on K_3 that are projected out by the action of g_i's. We will use these informations to determine the invariant components of the metric G_{MN} under compactification. The number of 2-form fields left invariant in the cases (i)-(iv) above are respectively 14, 10, 8 and 7. By subtracting these numbers from the dimension of the coset (5), and taking into account the rank-reduction in the last paragraph, we get the number of invariant scalars from the K_3 part, originating from the metric G_{MN} in ten dimesnions. These are respectively (i) 34, (ii) 22, (iii) 16 and (iv) 13.

We now first construct an orientifold of M-Theory for the action of symmetry g_1 and then obtain other models by additional projections with respect to the remaining g_i's. As mentioned earlier, to avoid fixed points with respect to these additional Z_2's, one has to combine them with the translations on circles [19, 20] by compactifying further to lower dimensions. However, since the field content for the massless modes does not depend on these shifts, these lower dimesnional spectra follow directly from the toroidal compactification of the six dimensional ones. Therefore the effect of these extra Z_2's can be seen directly in terms of a six dimensional spectrum which we now present. In the next four paragraphs, we present the field contents in the cases (i)-(iv) above, starting from the fields G_{MN} and A_{MNP} in eleven dimensions. We also show that they are all consistent with the anomaly cancellation requirements.

(i) The projection with respect to g_1 leaves the components $G_{\mu\nu}$ and $G_{(10)(10)}$ of G_{MN} invariant, and also gives 34 scalar fields from the local moduli in K_3. In addition, taking into account that A_{MNP} is odd under this Z_2 and x^{10} changes sign, we find that the component $A_{\mu\nu(10)}$ remains invariant and gives rise to an antisymmetric tensor field in six dimensions. We also get 8 gauge fields $A_{\mu mn}$ from the eight 2-forms of K_3 which are odd under g_1. 14 more scalars arise from even 2-form components $A_{mn(10)}$ on K_3. Together, these give us the graviton, an antisymmetric tensor, 8 vectors and 49 scalars in the untwisted sector[9]. Combining these with the states arising from the twisted sectors, namely 8 tensor and 8 hypermultiplets, we find the following final spectrum. (a) 9 tensor multiplets (b) 1 graviton multiplet (c) 8 vector multiplets and (d) 20 hypermultiplets. These are precisely the combination needed for the anomaly cancellatin in $N = 1$ supersymmetric theories[14, 9].

(ii) For this case the spectrum in the untwisted sector is obtained by taking intersection with respect to the projections in (i), as described in the last paragraph, with those with respect to g_2. The surviving degrees of freedom from G_{MN} now consists of $G_{\mu\nu}$, $G_{(10)(10)}$ and 22 local moduli fields. Among the A_{MNP} components, which survive this projection are now (a) $A_{\mu\nu(10)}$, (b) 4 of the eight vectors $A_{\mu mn}$ in the

last paragraph and (c) 10 of the fourteen scalars $A_{mn(10)}$. Together these provide a graviton, an antisymmetric tensor, 4 vectors and 33 scalars in the untwisted sector. Combining these, once again, with the fields from the twisted sectors, namely the 8 tensor and 8 hypermultiplets we get the full field content for this theory as: (a) 9 tensor multiplets, (b) a gravity multiplet, (c) 4 vector multiplets and (d) 16 hypermultiplets. This is once again an anomaly free spectrum[14]. As stated earlier, g_2 acts freely only when it is combined with a half-shift along one of the circles of compactification. For this purpose, one has to consider this orientifold only in dimensions five. However, since the field content for the massless fields does not depend on the shift, the resulting five dimensional fields can easily be seen to arise from the compactification of the above massless spectrum in six dimensions on a circle. This is a new anomaly free combination, other than the one discussed in [9].

(iii) The effect of the actions of g_2 and g_3 on the orientifold of g_1 in (i) can be studied exactly in the same way as in (ii) above. The massless fields from the untwisted sector now consist of a graviton, 1 antisymmetric tensor, 2 vector fields and 25 scalar fields. Once again, combining these with the 8 tensor and 8 hypermultiplets of the twisted sector gives rise to 9 tensor multiplets, a graviton multiplet, 2 vector multiplets and 14 hypermultiplets. This is an anomaly free field content. As in the last paragraph, the two g_i's have to be combined with appropriate shifts along the compactified directions. As a result, this time the model has a valid interpretation only in four dimensions. However once again, the resulting four dimesnional fields can be seen to originate from a six dimensional theory with the above anomaly free field content through a simple toroidal compactification.

(iv) Finally, when all the four g_i's are applied together, with g_1 acting as an orientifold as above, we have from the untwisted sector, a graviton, 1 antisymmetric tensor, 1 vector field and 21 scalars. Combining them with the twisted sector states, i.e. 8 tensor and 8 hypermultiplets, we get another anomaly free field content: 9 tensor multiplets, 1 gravity multiplet, 1 vector multiplet and 13 hypermultiplets.

To conclude, we have presented new anomaly free combinations in six dimensions that arise from the $K_3 \times S^1$ orientifold of M-Theory by additional projections with respect to the abelian automorphism groups of K_3. One of the key ingredients in constructing these $N = 1$ supersymmetric theories has been the fact that the number of supersymmetries do not reduce by the above actions of the automorphisms of K_3. We have also avoided the presence of any new twisted sector states by combining the extra Z_2 projections with the shifts along the directions on which the six dimesnional theory is compactified. Another way to avoid additional fixed points may be by combining these additional Z_2's with the shifts along the central charges of the supersymmetry algebra[21]. We will then have a genuinely six dimensional theory with the same field contents as mentioned in this paper. One may also directly study the contributions from all the additional fixed points, which have been avoided here, and see if the anomaly cancellation condition is maintained.

References

[1] E. Witten, “String Theory Dynamics in Various Dimensions”, *Nucl. Phys.* **B 443** (1995) 85.

[2] J.H.Schwarz, “The Power of M Theory”, **hep-th /9510086**.

[3] J. Maharana, “M Theory and p-branes”, **hep-th /9511159**.

[4] P. Horava and E. Witten, ”Heterotic and Type I String Dynamics from Eleven Dimensions”, **hep-th/9510209**.

[5] K. Dasgupta and S. Mukhi, “Orbifolds of M-Theory”, **hep-th /9512196**.

[6] E. Witten, “Five-branes and M Theory on an Orbifold”, **hep-th /9512219**.

[7] J.H. Schwarz, *M Theory Extensions of T-Duality*, **hep-th /9601077**.

[8] K. Davis, “M-Theory and String-String Duality”, **hep-th/9601102**.

[9] A. Sen, “M-Theory on $K3 \times S^1/Z_2$”, **hep-th /9602010**.

[10] A. Sagnotti, “Open Strings and their Symmetry Groups”, in *Non-perturbative Quantum Field Theory*, Cargese 1987, eds. G. Mack et. al. (Pergamon Press 1988); P. Horava, “String on Worldsheet Orbifolds”, *Nucl. Phys.* **B 327** (1989) 461; “Background Duality of Open String Models”, *Phys. Lett.* **B 321** (1989) 251.

[11] J. Dai, R. Leigh, and J. Polchinski, “New Connections between String Theories”, *Mod. Phys. Lett.* **A4** (1989) 2073.

[12] J. Polchinski, “Dirichlet-Branes and Ramond-Ramond Charges”, **hep-th/9510017**.

[13] L. Alvarez-Gaume’ and E. Witten, “Gravitational Anomalies”, *Nucl. Phys.* **B 234** (1984) 269.

[14] P. Townsend, *Phys. Lett.* **B 139** (1984) 283.

[15] C. Vafa and E. Witten, "Dual String Pairs with N=1 and N=2 Supersymmetry in Four Dimensions", *Nucl. Phys.* **B** (1995) , hep-th/9507050.

[16] A. Kumar, "Orientifold and Type II Dual Pairs", **hep-th/9601067**.

[17] A. Dabholkar and J. Park, "An Orientifold of Type IIB Theory on K3", **hep-th/9602030**.

[18] M. Walton, "Heterotic String on simplest Calabi-Yau manifold and its orbifold limits", *Phy. Rev.* **D37** (1988) 377.

[19] J. Schwarz and A. Sen, "The type IIA dual of the six Dimesnsional CHL compactification", **hep-th/9507027**.

[20] S. Chaudhuri and D. Lowe, "Type IIA-Heterotic Duals with Maximal Supersymmetry", **hep-th/9508144**.

[21] A. Sen and C. Vafa, "Dual Pairs of Type II String Compactification", **hep-th/9508064**.